

1. Consider the Battery Data which consists of 16 observations with a balanced two-way ANOVA design. The two factors are “Brand” with two levels (“Name” and “Store”) and “Duty” with two levels (“Alkaline” and “Heavy”). Suppose we have the following R output from the following linear model fit.

```
>options(contrasts=c("contr.sum", "contr.sum"))
>summary(lm(Cost~Brand*Duty, data=battery))
Call:
lm(formula = Cost ~ Brand * Duty, data = battery)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   590.125      ?????? ?????????? ?????????? ***
Brand1        -88.250      ?????? ?????????? ?????????? ***
Duty1         125.500      ?????? ?????????? ?????????? ***
Brand1:Duty1  -56.625      ?????? ?????????? ?????????? ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 48.659 on 12 degrees of freedom
Multiple R-squared:  0.93774, Adjusted R-squared:  0.92217
F-statistic: 60.243 on 3 and 12 DF,  p-value: 1.6624e-07
```

Please answer the following questions based on the above output.

- (a) (20 points) The group average of “Name” and “Heavy” is 433, and that of “Name” and “Alkaline” is 570.75. Find the other two group averages.
  - (b) (25 points) Find the standard errors of the four estimates in the R output.
2. (25 points) Suppose  $Y_i | \mathbf{X}_i \sim \text{Binomial}(n_i, \pi_i)$ ,  $i = 1, \dots, I$ , and each observation has the same weight. Suppose the GLM with the logit link provides estimates  $\hat{\pi}_i$ 's for  $\pi_i$ 's. Derive the scaled deviance.
  3. (30 points) Consider the following Poisson-Gamma model

$$f(y|z) = \frac{z^y}{y!} e^{-z}, \quad f(z|\mu, \theta) = \frac{\theta^\theta}{\Gamma(\theta)} \frac{z^{\theta-1}}{\mu^\theta} e^{-\frac{\theta z}{\mu}}.$$

Suppose  $\theta$  is known. Consider the MLE of  $\mu$ . Find the E-step, M-step and the updating rule in the EM algorithm which treats  $Z$  as the missing data.