1. In 23 space shuttle missions, some evidence of damage due to blow by and erosion was recorded on some O-rings. For each mission, we know the number of O-rings out of 6 showing some damage and the launch temperature. The output of a logistic regression is shown below.

```
# Call:
# glm(formula = cbind(damage, 6 - damage) ~ temp, family = binomial,
# data = orings)

# Coefficients:
            # Estimate Std. Error z value Pr(>|z|)
# (Intercept) 11.66299    3.29626   3.538 0.000403 ***
# temp        -0.21623    0.05318  -4.066 4.78e-05 ***
# ---
# Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘’ 1

# (Dispersion parameter for binomial family taken to be 1)
```

Please answer the following questions based on the above output.

(a) (15 points) Find the predicted probability of damage when the temperature is 31.

(b) (15 points) Find the (multiplicative) change in odds when the temperature changes from 79 to 69.

(c) (15 points) Find the Pearson residual of the observation with 1 damaged O-ring and temperature 75.

(d) (15 points) Find the deviance residual of the observation with 5 damaged O-rings and temperature 53.

2. (20 points) Consider n samples from Gamma distributions whose density functions are

$$f(x_i|\mu_i,\nu) = \frac{1}{\Gamma(\nu)}\left(\frac{\nu}{\mu_i}\right)^{\nu} x_i^{\nu-1} e^{-\frac{x_i\nu}{\mu_i}}, \qquad \mu_i > 0$$

for $x_i \in (0,\infty), i = 1,\ldots,n$ and known $\nu > 0$. Find the quasi-likelihood function Q.

3. (20 points) Consider the following random effect model:

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij} \tag{1}$$

with $i = 1,\ldots,I$ and $j = 1,\ldots,J$. It is assumed that $\epsilon_{ij} \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$ and are independent of $\alpha_i \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\alpha^2)$. Find the E-step and M-step in the EM algorithm for the MLE of $(\mu,\sigma_\epsilon^2,\sigma_\alpha^2)$.