Or006: Assignment #2 Solutions

L=long-legged, l=short-legged, P=pointy-eared, p=flap-eared

1.

"long-legged, flap-eared" dog: $(L + L)(p + p) = 4Lp$

"short-legged, pointy-eared" dog: : $(l + l)(P + P) = 4lP$

offspring polynomial: $16LlPp$ (all "long-legged, pointy-eared")

2. genotype polynomial for the next offspring:

$$\begin{aligned}&(L + l)(P + p)(L + l)(P + p)\\ &\quad= (LP + lP + Lp + lp)(LP + lP + Lp + lp)\\ &\quad= LLPP + 2LlPP + llPP + 2LLPp + 4LlPp + 2llPp + llPP + 2llPp + llpp\end{aligned}$$

Divide this by 16=sum of all coefficients:

$$\frac{1}{16}LLPP+\frac{2}{16}LlPP+\frac{1}{16}llPP+\frac{2}{16}LLPp+\frac{4}{16}LlPp+\frac{2}{16}llPp+\frac{1}{16}LLpp+\frac{2}{16}Llpp+\frac{1}{16}llpp$$

phenotypes: $\frac{9}{16}$ "long-legged pointy-eared", $\frac{3}{16}$ "short-legged pointy-eared", $\frac{3}{16}$ "long-legged flap-eared", and $\frac{1}{16}$ "short-legged flap-eared".

3. Offspring of a population of $\frac{2}{10}LLpp$ and $\frac{8}{10}llPP$:

$$\begin{aligned}&\tfrac{2}{10} \times \tfrac{2}{10} \times \tfrac{1}{16} \times (L + L)(p + p)(L + L)(p + p) = \tfrac{1}{25} \times LLpp\\ &+ 2 \times \tfrac{2}{10} \times \tfrac{8}{10} \times \tfrac{1}{16} \times (L + L)(p + p)(l + l)(P + P) = \tfrac{8}{25} \times LlPp\\ &+ \tfrac{8}{10} \times \tfrac{8}{10} \times \tfrac{1}{16}(l + l)(P + P)(l + l)(P + P) = \tfrac{16}{25} \times llPP\end{aligned}$$

Totals: 4% "long-legged, flap-eared" (all $LLpp$), 32% "long-legged, pointy-eared" (all $LlPp$), and 64% "short-legged, pointy-eared" (all $llPP$).

4. Around Generation 25 we get

genotype	$LLPP$	$LlPP$	$llPP$	$LLPp$	$LlPp$	$llPp$	$LLpp$	$Llpp$	$llpp$
fraction	16/625	128/625	256/625	8/625	64/625	128/625	1/625	8/625	16/625

phenotypes: 216/625 "long-legged, pointy-eared", 384/625 "short-legged, pointy-eared", 9/625 "long-legged, flap-eared", and 16/625 "short-legged, flap-eared".

5. Multiplying Darwin factors to Problem 2, we have

"long-legged, pointy-eared" ($LLPP$, $LlPP$, $LLPp$, $LlPp$) gets a multiple of $5 \times 2 = 10$.

"short-legged, pointy-eared" ($llPP$, $llPp$) gets a multiple of $1 \times 2 = 2$.

"long-legged, flap-eared" ($LLpp$, $Llpp$) gets a multiple of $5 \times 4 = 20$.

"short-legged, flap-eared" ($llpp$) gets a multiple of $1 \times 4 = 4$.

The whole polynomial is

$$\begin{aligned}&10 \times \tfrac{1}{16} LLPP + 10 \times \tfrac{2}{16} LlPP + 2 \times \tfrac{1}{16} llPP + 10 \times \tfrac{2}{16} LLPp \\ &\quad + 10 \times \tfrac{4}{16} LlPp + 2 \times \tfrac{2}{16} llPp + 20 \times \tfrac{1}{16} LLpp + 20 \times \tfrac{2}{16} Llpp + 4 \times \tfrac{1}{16} llpp \\ &= \tfrac{10}{16} LLPP + \tfrac{20}{16} LlPP + \tfrac{2}{16} llPP + \tfrac{20}{16} LLPp \\ &\quad + \tfrac{40}{16} LlPp + \tfrac{4}{16} llPp + \tfrac{20}{16} LLpp + \tfrac{40}{16} Llpp + \tfrac{4}{16} llpp\end{aligned}$$

The coefficients sum to $\frac{160}{16} = 10$. Dividing by 10, or equivalently, replacing the value of 16 in the denominator by 160, we get the correct genotype proportions, and a phenotype population of $\frac{90}{160} = 56.25\%$ "long-legged, pointy-eared", $\frac{6}{160} = 3.75\%$ "short-legged, pointy-eared", $\frac{60}{160} = 37.5\%$ "long-legged, flap-eared", and $\frac{4}{160} = 2.5\%$ "short-legged, flap-eared".

6. After 25 generations, it looks like everything but *LLpp* (about 90%) and *Llpp* (about 10%) has died out. (Actually, the *Llpp*'s will die out after a very long time.) In any case, there will be nothing but "long-legged, flap-eared" dogs left.