1. State and prove the version of the weak law of large numbers presented in class.

2. Suppose that \( Y_1, \ldots, Y_n \) are random variables such that \( E e^{sY_i} \leq e^{s^2/2} \) for each \( s \geq 0 \). Show that
\[
\Theta = E \left[ \max_{1 \leq i \leq n} Y_i \right] \leq \sqrt{2 \ln n}
\]
Hint: Begin by considering \( e^{s\Theta} \) for a value \( s > 0 \).

3. Let \( X_1, X_2, \ldots, X \) be random variables, all defined on the same probability space, such that \( X_n \) converges to \( X \) in probability. Establish the following relations directly, without appealing to results from class.
   a. Show that \( X = O_P(1) \).
   b. Show that \( X_n = O_P(1) \).
   c. Show that \( O_P(1) \cdot o_P(1) = o_P(1) \).

4. Let \( X_1, X_2, \ldots, X \) be identically distributed non-negative random variables such that \( EX^2 \) is finite. Note that the \( X_i \) may be dependent.
   a. Show that \( EXI\{X > \alpha\} \to 0 \) as \( \alpha \to \infty \).
   b. Show carefully that for \( \alpha > 0 \),
\[
\max_{1 \leq i \leq n} X_i \cdot I \left\{ \max_{1 \leq i \leq n} X_i \geq \alpha \right\} \leq \max_{1 \leq i \leq n} X_i \cdot I\{X_i \geq \alpha\}
\]
   c. Show that
\[
n^{-1} E \left[ \max_{1 \leq i \leq n} X_i \right] \to 0 \quad \text{as} \quad n \to \infty.
\]

5. For \( n \geq 1 \) let \( X_n \) have a Bern\((n,p)\) distribution, where \( p \in (0,1) \) is fixed. What can you say about the limiting distribution of the random vectors \( Y_n = (X_n, n - X_n)^T \) after suitable scaling and/or shifting?
6. Let $X_1, \ldots, X_n$ be an i.i.d. sample from a population with $E X = \mu$ and $\text{Var}(X) = \sigma^2 < \infty$. Let $\bar{X}_n = n^{-1} \sum_{i=1}^{n} X_i$ be the sample mean of $X_1, \ldots, X_n$.

a. After suitable scaling and/or shifting, what can you say about the limiting distribution of $(\bar{X}_n)^3$ when $\mu \neq 0$?

b. After suitable scaling and/or shifting, what can you say about the limiting distribution of $(\bar{X}_n)^3$ when $\mu = 0$?

7. Let $X, Y$ be random variables such that $E X^2$ and $E Y^2$ are finite. Let $SD(\cdot)$ denote standard deviation. Show that $SD(X + Y) \leq SD(X) + SD(Y)$.

8. If $X \sim \mathcal{N}_p(\mu, \Sigma)$, find the distribution of $AX + b$ where $A$ is a $d \times p$ matrix and $b \in \mathbb{R}^d$. 