1. In this question, we shall focus on the geometric distribution $G(\theta)$, which has a probability density function
$$p(x|\theta) = \theta(1-\theta)^{x-1}$$
for $\theta \in (0,1)$. Suppose we observe i.i.d. samples $X_1, \dots, X_n$.
   (a) Show that $G(\theta)$ an exponential family.
   (b) Find the MLE, T_n, for $\frac{1}{\theta}$, as well as its expectation and variance.
   (c) Find the variance stabilizing function $g(\cdot)$ such that $\sqrt{n}(g(T_n) - g(\frac{1}{\theta})) \to \mathcal{N}(0,1)$.
   (d) Let $P = G(\theta_1)$ and $Q = G(\theta_2)$. Find the Hellinger affinity $\rho(P,Q)$.

2. Suppose $X_1, \dots, X_n \overset{i.i.d.}{\sim} Uniform[0,1]$. Find the limiting distribution of $M_n = n \min_{1\le i \le n} X_i$ as $n \to \infty$.

3. Suppose the sample space is $S = \{0,1\}$, the parameter space is $\Theta = \{0,1\}$, and the loss function is the squared error loss. Consider the problem of using one observation $X \in S$ to estimate $\theta \in \Theta$. Is a minimax estimator of θ always admissible? Provide a proof or a counterexample.

4. (A Consistency Statement of the MLE)
Let $X_1, \dots, X_n$ be independently sampled from a density $p(x|\theta_0)$ for $\theta_0 \in \Theta$. The likelihood function is
$$L(\theta) = \prod_{i=1}^{n} p(X_i|\theta).$$
Suppose the parametrization is identifiable. Show that for any $\theta \neq \theta_0$, as $n \to \infty$.
$$\mathbf{P}\left(\frac{L(\theta_0)}{L(\theta)} > 1\right) \to 1.$$
Hint: Use the Shannon-Kolmogorov inequality.

5. Consider a kernel $K(x) = (a + bx^2)I(|x| \le 1)$ in density estimation. Find constants a and b such that $K(x)$ is of order 3.