COMPREHENSIVE WRITTEN EXAMINATION
Stat 154, August 2005

Problem 1:

(a) State precisely

(i) The Monotone Convergence Theorem

(ii) The Dominated Convergence Theorem

(iii) Fatou's Lemma

(b) Let f_n be integrable functions on a measure space $(X, \mathcal{S}, \mu)$ such that $sup \int f_n d\mu < \infty$. If $f_n(x) \uparrow f(x)$ a.e. where f is measurable, show that f is integrable and $\int f_n d\mu \to \int f d\mu$.

(c)Let f_n, h_n, f, h be integrable functions on a measure space $(X, \mathcal{S}, \mu)$ with $f_n \to f$ a.e., $h_n \to h$ a.e. and $\int f_n d\mu \to \int f d\mu$, $\int h_n d\mu \to \int h d\mu$. If also g_n, g are measurable, $g_n \to g$ a.e. and $f_n \leq g_n \leq h_n$ a.e., show that g_n and g are integrable and $\int g_n d\mu \to \int g d\mu$.

Problem 2:

(a) State (precisely) two forms of Fubini's Theorem.

(b) If $f(x,y) = \frac{x^2-y^2}{(x^2+y^2)^2}$, evaluate $\int_0^1 (\int_0^1 f(x,y)dy)dx$ and $\int_0^1 (\int_0^1 f(x,y)dx)dy$ and comment regarding what these say about Fubini's Theorem.