STAT 155 Problem

Problem 1:

(a) Define:

(i) the distribution function of a random variable

(ii) the mean of a random variable in two ways and show their equivalence

(b) Show that if two random variables, ξ, η have the same distribution function, F, then $P\{\xi \in B\} = P\{\eta \in B\}$ for each Borel set B.

(c) If ξ, η are random variables defined on possibly different probability spaces, show how to define a new probability space and independent random variables ξ^*, η^* with the same distributions as ξ, η respectively.

Problem 2:

(i) Define the term "characteristic function" $\phi(t)$ of a random variable ξ.

(ii) If ξ is absolutely continuous with pdf f and it's characteristic function $\phi(t) \in L_1(-\infty, \infty)$, state an inversion formula giving f in terms of ϕ.

(iii) Show that the charateristic function of "Laplace's distribution" with pdf $f(x) = \frac{a}{2}e^{-a|x|}$ is $\phi(t) = a^2/(a^2 + t^2)$.

(iv) Use your inversion formula to obtain the charateristic function of the Cauchy distibution with pdf $1/[\pi(1 + x^2)]$.

(v) Show that if ξ, η are independent random variables, then the characteristic function of $(\xi + \eta)$ is the product of the characteristic functions of ξ and η, but that this can also happen even if ξ, η are not independent (e.g. try $\xi = \eta$ with an appropriate pdf).